TITLE: Fatigue Assessment of an Exhaust System using Antiresonance Frequencies

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ABSTRACT

Antiresonances have become an attractive alternative in structural damage assessment. They can be identified easier and more accurately than mode shapes, and still providing the same information. Antiresonances are derived from point frequency response functions (FRFs) or from transfer FRFs. However, antiresonances from transfer FRFs are very sensitive to small structural changes, and the matching between numerical and experimental antiresonances is affected. This problem is solved if antiresonances from point FRFs are used. However, it implies an experimental procedure that differs from a common modal testing, which may become not practical or too expensive. This paper proposes a damage detection method able to deal with transfer antiresonances. The inverse problem is handled by a Parallel Genetic Algorithm. In this case, a perfect match between the antiresonances is not required because the optimization is not gradient based. Moreover, the matching can change at each step and the optimization is not affected. An exhaust system of a car with a single fatigue crack is used to verify the approach; three increasing levels of damage are studied. Damage detected is consistent with the experimental damage.

1 INTRODUCTION

Recently, greatly attention has been given to the possible use of antiresonances in structural damage detection. Antiresonances are an attractive alternative because they can be determined easier and with less error than mode shapes. Wahl et al. [1] study the possibilities of antiresonances as indicators of structural modifications. They conclude that antiresonances may lead to future applications of identification and location of structural faults, although no implementation is attempted. Mottershead [2] shows that antiresonances sensitivities are linear combination of eigenvalues and mode shapes sensitivities. Hence, antiresonances are an alternative to mode shapes since they provide the same information. As natural frequencies, antiresonances are located along the frequency axis and can be estimated from experimental FRFs more accurately than mode shapes. Antiresonances are also very

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sensitive to small structural changes, which makes them good damage indicators. Despite these advantages, the use of antiresonances it still under development and the application of antiresonances to structural damage detection has not been fully investigated.

Antiresonances can be derived from point FRFs, where the response coordinate is the same as the excitation coordinate; or from transfer FRFs, where the response coordinate differs from the excitation coordinate. Point FRFs are preferred because matching problems arise when antiresonances from transfer FRFs are used [3]. However, the procedure to obtain point FRFs differs from common modal testing, i.e. the excitation degree of freedom (DOF) is moved together with the response DOF. This may become not practical or too expensive.

This work intends to use transfer antiresonances and natural frequencies to solve the inverse problem of model based damage assessment. The optimization is handled by a Parallel Genetic Algorithm (PGA). In this case, a perfect match between the antiresonances is not required because the optimization is not gradient based. Moreover, the matching can change at each step and the optimization is not affected. To ensure a balance search, each population uses a different crossover and two mutation operators. The damage penalization proposed by Meruane and Heylen [4, 5] is used to successfully avoid false damage detection. An automated selection of the penalization parameter is implemented. The damage assessment algorithm is applied to an exhaust system of a car with a single fatigue crack. The crack is introduced by a fatigue test; three increasing levels of damage are studied.

2 FORMULATION OF THE OPTIMIZATION PROBLEM

Damage is represented by an elemental stiffness reduction factor β_i, defined as the ratio between the stiffness reduction to the initial stiffness. The value $\beta_i=0$ indicates that the element is undamaged whereas $0< \beta_i \leq 1$ implies partial or complete damage. The objective function correlates transfer antiresonances and natural frequencies. To avoid the need of an accurate numerical model, its initial correlation is included to the objective function. The error in natural frequencies is represented by the ratio between the experimental and analytical eigenvalues,

$$\varepsilon_{\lambda,i}(\{\beta\}) = \frac{\lambda_{A,i}(\{\beta\})}{\lambda_{E,i}} - \frac{\lambda_{A0,i}}{\lambda_{E0,i}} = \frac{\omega_{A,i}^2(\{\beta\})}{\omega_{E,i}^2} - \frac{\omega_{A0,i}^2}{\omega_{E0,i}^2} \tag{1}$$

The subscripts A and E refer to analytical and experimental respectively and the subscript 0 refers to the initial undamaged state. λ_i is the *i*th eigenvalue and ω_i is the *i*th natural frequency. The error in antiresonances is represented by the ratio between the experimental and analytical antiresonances

$$\varepsilon_{r_r i,n}(\{\beta\}) = \frac{\omega_{r,i,n}^{A}(\{\beta\})^2}{\omega_{r,i,n}^{E}{}^2} - \frac{\omega_{r,i,n}^{A0}{}^2}{\omega_{r,i,n}^{E0}{}^2} \tag{2}$$

$\omega_{r,i,n}$ is the *i*th antiresonance of the *n*th FRF. In equations (1) and (2) the goal is not to reach a perfect match between the numerical and experimental parameters, but rather to reach the same correlation than in the undamaged case. This considers initial errors in the numerical model. If a damaged mode or antiresonance cannot be matched with an undamaged one; its initial correlation is set equal to 1. This occurs in cases where after the introduction of damage, new modes or antiresonances are detected. The objective functions are the normalized sum of the errors plus a damage penalization term,

$$J(\{\beta\}) = \frac{F_\lambda(\{\beta\})}{F_{\lambda,0}} + \frac{F_{\omega_r}(\{\beta\})}{F_{\omega_r,0}} + F_D(\{\beta\})$$
$$F_\lambda(\{\beta\}) = \sum_i \left\| \varepsilon_{\lambda,i}(\{\beta\}) \right\| \tag{3}$$
$$F_{\omega_r}(\{\beta\}) = \sum_n \sum_i \left\| \varepsilon_{r,i,n}(\{\beta\}) \right\|$$

$F_{\lambda,0}$ and $F_{\omega r,0}$ refers to the initial values of the sums (β=0). F_D is a damage penalization function. Damage penalization helps to avoid false damage detection because of experimental noise or numerical errors [4, 5]. Two damage penalization functions are used:

$$F_{D,1} = \gamma_1 \sum_i \beta_i$$
$$F_{D,2} = \gamma_2 \sum_i \delta_i, \quad \delta_i = \begin{cases} 1 & \beta_i > 0 \\ 0 & \beta_i = 0 \end{cases} \tag{4}$$

The first penalizes the total amount of damage. The second, on the other hand, penalizes the number of damage locations. Depending on the damage pattern expected one can use the first function, the second or a combination of both. The value of the constants γ_1 and γ_2 depend on the confidence in the numerical model and the experimental data.

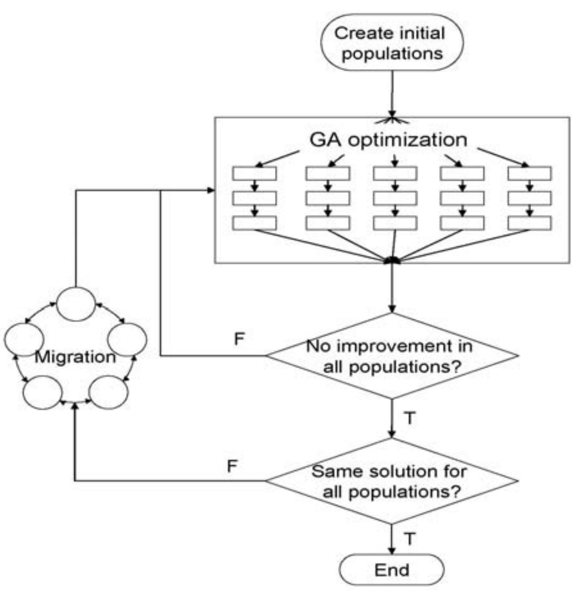

Figure 1 Parallel optimization

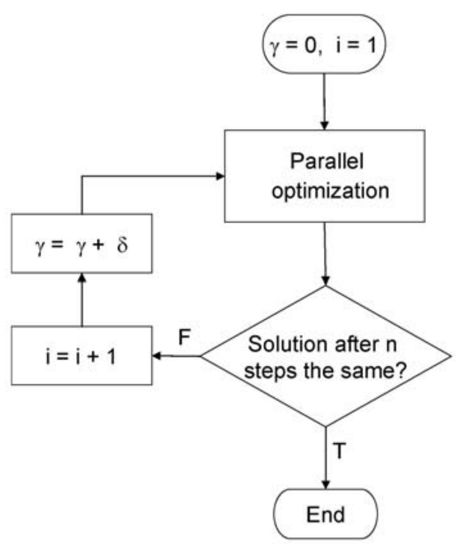

Figure 2 Damage detection algorithm

3 DAMAGE DETECTION ALGORITHM

The algorithm employs a multiple population GA with five populations and a neighborhood migration. The penalization function selected is the sum of $F_{D,1}$ and $F_{D,2}$ (see equation (6)) with $\gamma_1=\gamma_2=\gamma$. A normalized geometric selection is used as was recommended by Meruane and Heylen [4]. To ensure an effective search with an adequate balance between exploration and exploitation, each population works with a different crossover operator. In addition, each population applies both boundary and uniform mutations. Each population has a size of 40 individuals and the crossover and mutation probabilities are: p_c=0.80 and p_m=0.02 respectively. The migration interval is automatically adjusted. If a population has no improvement after a predefined number of generations, the GA stops and exchanges the individuals with their neighbors. This exchange of individuals is synchronous i.e., the algorithm waits until the five populations are ready to perform the migration. At each migration, each population sends its best individual, whereas its worse individual is replaced by the received individual. Before each migration, the best individuals from all populations are compared, if they are all the same the optimization is finished. Figure 1 illustrates this process.

Because the appropriate value for γ it is not known, its value is dynamically adjusted as shown in Figure 2. First the solution with γ=0 is computed, next the value of γ is increased by δ and the solution is recomputed. This process is iterated until a stable solution is reached and stops. The solution is defined stable if after three consecutive steps it remains the same. The value of δ used is 0.02.

4 EXPERIMENTAL CASE

The structure is a car exhaust system as shown in Figure 3. The dimensions are: length: 2.3m, width: 0.45m. The exhaust pipe has a diameter of 38mm. The structure is suspended by soft springs and it is excited randomly by an electrodynamic shaker. The response is captured by 20 accelerometers. The test is performed in a frequency range of 0-1024Hz with a frequency resolution of 0.25Hz.

The numerical model shown in Figure 5 is built in Matlab with 2D beam element and concentrated inertias for the masses. The model has 47 beam and 3 inertia elements, with 144 DOF. The minimum MAC value between the numerical model and the experimental data is 0.98 and the maximum frequency difference is 2.78%.

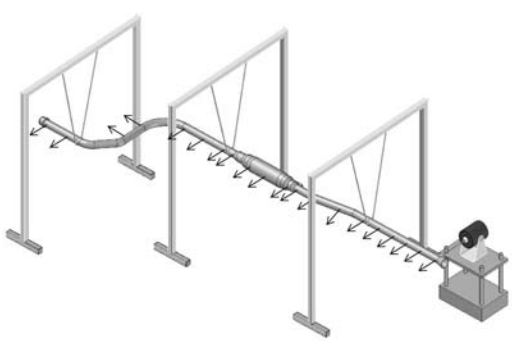

Figure 3 Experimental set-up

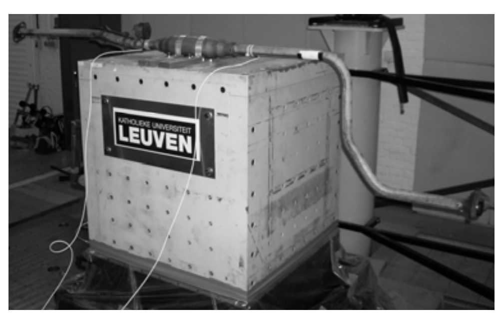

Figure 4 Fatigue test to introduce damage to the exhaust system

A single fatigue crack with three increasing levels of damage is introduced to the structure. To develop the fatigue crack, the structure is clamped over a 6-DOF shaker table as shown in Figure 4. The shaker excites the structure in the vertical direction with frequencies from 0 to 50Hz. The first two modes of the clamped structure are excited, therefore inducing large displacement vibrations. The largest strain is located at the welded connections between the resonator and exhaust pipe. Visual observations and strain gauge readings are used during the test to detect the presence of damage. The fatigue test continues until a visible macro-crack is observed. Figure 6-a) shows the crack, this crack is located in the welded connection between elements 30 and 31 (see Figure 5) and covers around 60% of the pipe perimeter. The fatigue test is done again twice to grow the crack. Figure 6-b) shows the second damage level; here the structure has already failed due to unstable crack propagation. The open crack covers around 70% of the perimeter. The last damage level is shown in Figure 6-c), the crack covers around 85% of the perimeter. After each crack size, the structure is subjected to an experimental modal analysis.

Damage is detected through the algorithm described in section 3. The 47 beam elements are considered as possible damage locations. The first natural frequencies are used. All the identified antiresonances in the range 0-300Hz from the 20 measured FRFs are used. The number of identified antiresonances varies from 26 to 34 depending on the case. Figure 7 show the results, if γ equals zero a significant amount of false damage is detected. This false damage is sequentially reduced by increasing the value of γ until a stable solution is reached. Once the stable solution is reached the algorithm stops. The experimental crack is located between elements 30 and 31, thus we expect to detect damage in one of these two elements. In the first case, the experimental damage is successfully detected in element 31 with a 90% of damage, although false damage is still detected in elements 33 and 34. In the second and third cases, the approach detects damage in element 32 with a 92% of damage.

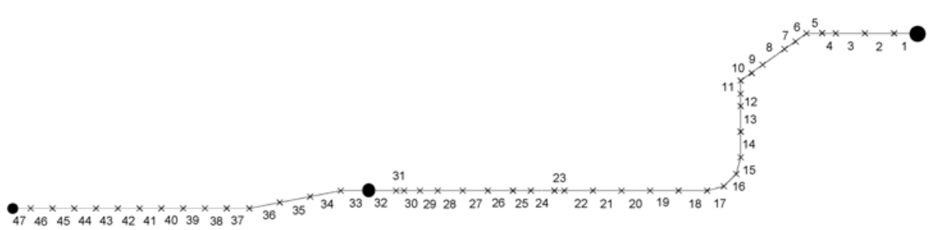

Figure 5 Finite element model and element numbering

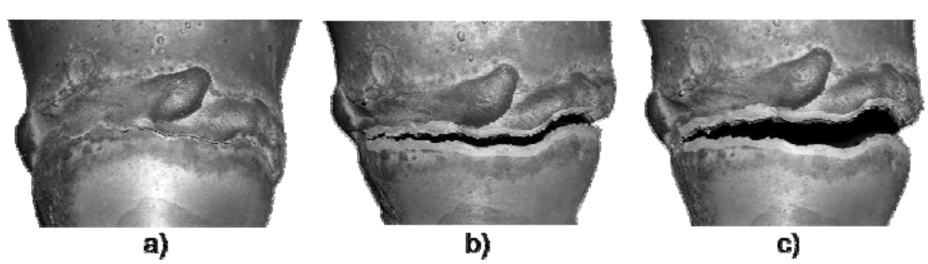

Figure 6 Three levels of damage introduced to the structure

5 CONCLUSIONS

A damage detection method using transfer antiresonances and Parallel Genetic Algorithms have been implemented. False damage detection is avoided by damage penalization. The algorithm makes an automated selection of the penalization parameter. The objective function includes the initial errors in the numerical model; hence it is not necessary to start from a very accurate numerical model. The damage assessment algorithm is applied an exhaust system of a car with a single fatigue crack. The damage detected has a good correspondence with the experimental damage. The algorithm was successful on using antiresonances from transfer FRFs to detect structural damage. Hence Parallel Genetic Algorithms are a good solution to handle this difficult optimization problem. Further studies in the identification, selection and matching of the antiresonances should be conducted to improved the damage assessment results.

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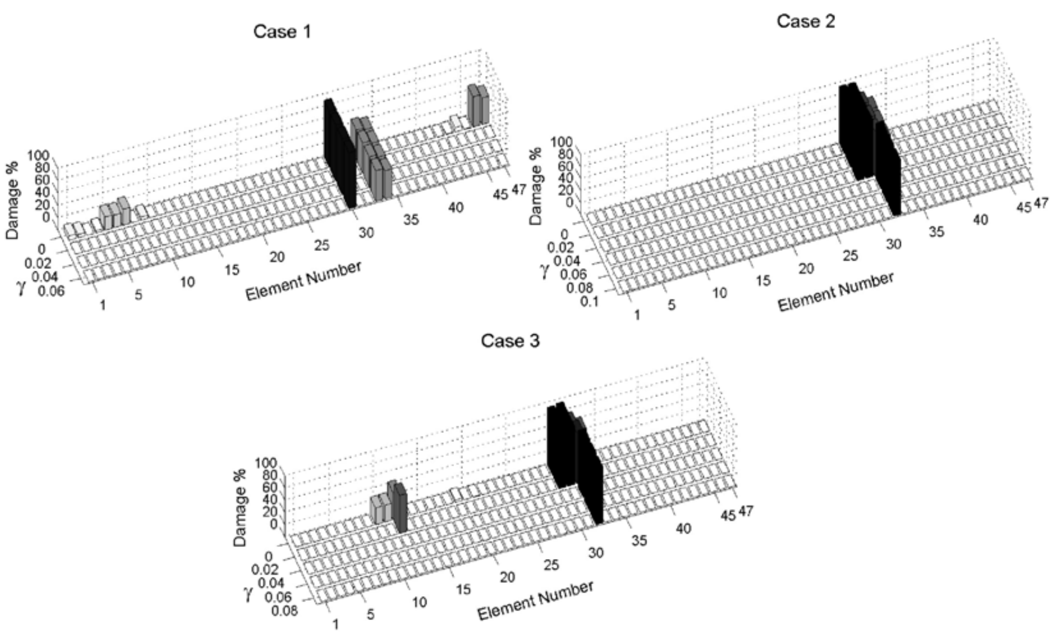

Figure 7 Damage assessment results